

① Za svaku Hermitovu matricu A reda n postoji unitarna matrica U takva da je U^*AU dijagonalna.

$$U^*AU = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_n \end{pmatrix}$$

② Za svaku matricu A reda n postoji unitarna matrica U takva da je U^*AU gornje trougaona matrica

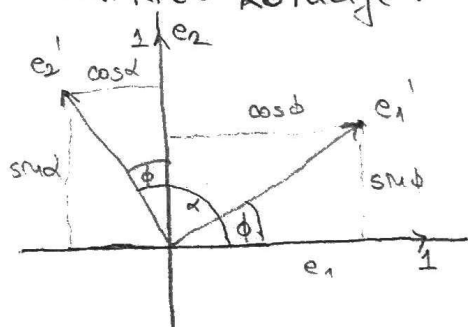
$$U^*AU = \begin{pmatrix} \lambda_1 & * \\ 0 & \lambda_n \end{pmatrix}$$

Jakobi, QR, ... \rightarrow određivanje $\lambda_i(A)$

Givens, Householder \rightarrow transformacija u gornje-Hessenbergovu $A \sim \begin{pmatrix} * & * & \dots & * \\ * & * & \dots & * \\ 0 & * & \dots & * \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & * \end{pmatrix}$

ili trodijagonalnu ako je A Hermitova $A \sim \begin{pmatrix} * & * & \dots & * \\ * & * & \dots & * \\ 0 & * & \dots & * \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & * \end{pmatrix}$

Matrice rotacije:



$$e_1 \rightarrow e_1', \quad e_2 \rightarrow e_2'$$

$$[1, 0] \rightarrow [\cos \phi, \sin \phi], \quad [0, 1] \rightarrow [-\sin \phi, \cos \phi]$$

$$\cos \alpha = \cos\left(\frac{\pi}{2} + \phi\right) = \cos \frac{\pi}{2} \cdot \cos \phi - \sin \frac{\pi}{2} \cdot \sin \phi = -\sin \phi$$

$$\sin \alpha = \sin\left(\frac{\pi}{2} + \phi\right) = \cos \frac{\pi}{2} \cdot \sin \phi + \sin \frac{\pi}{2} \cdot \cos \phi = \cos \phi$$

$$U = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix}$$

$$U \cdot e_1 = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos \phi \\ \sin \phi \end{bmatrix}$$

$$U \cdot e_2 = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \alpha \\ \sin \alpha \end{bmatrix} = \begin{bmatrix} -\sin \phi \\ \cos \phi \end{bmatrix}$$

$$U = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix}$$

unitarna

GIVENSOVA METODA

$A \rightarrow B$, B -gornjelless., $A \sim B$

$$U_{kl} = \begin{pmatrix} 1 & & & 0 \\ & \cos \phi & & -e^{i\psi} \sin \phi \\ & e^{i\psi} \sin \phi & & \cos \phi \\ 0 & & & 1 \end{pmatrix} \begin{matrix} \leftarrow k \\ \leftarrow l \\ \uparrow k \\ \uparrow l \end{matrix}$$

$$U = \begin{pmatrix} 1 & & & 0 \\ & \alpha & & -\bar{\beta} \\ & & \ddots & \\ 0 & \beta & & \alpha \end{pmatrix}$$

Za vektor $[x_1, x_2]^T \rightarrow [y_1, 0]^T$

$$\begin{aligned} A &= (a_{ij}), \quad B = (b_{ij}) = A \cdot U_{kl} \\ C &= (c_{ij}) = U_{kl}^* \cdot B = U_{kl}^* A \cdot U_{kl} \end{aligned} \quad \left. \vphantom{\begin{aligned} A &= (a_{ij}), \quad B = (b_{ij}) = A \cdot U_{kl} \\ C &= (c_{ij}) = U_{kl}^* \cdot B = U_{kl}^* A \cdot U_{kl} \end{aligned}} \right\} A \sim C$$

$$B = A \cdot \begin{pmatrix} 1 & & & 0 \\ & \alpha & & -\bar{\beta} \\ & & \ddots & \\ 0 & \beta & & \alpha \end{pmatrix}$$

$$\begin{aligned} b_{ik} &= a_{ik} \cdot \alpha + a_{il} \cdot \beta \\ b_{il} &= -a_{ik} \cdot \bar{\beta} + a_{il} \cdot \alpha \\ \Rightarrow b_{ij} &= a_{ij}, \quad j \neq k, l \end{aligned} \quad \left. \vphantom{\begin{aligned} b_{ik} &= a_{ik} \cdot \alpha + a_{il} \cdot \beta \\ b_{il} &= -a_{ik} \cdot \bar{\beta} + a_{il} \cdot \alpha \end{aligned}} \right\} \begin{array}{l} \text{menjaju se elementi} \\ k\text{-te i } l\text{-te kolone} \end{array}$$

$$C = \begin{pmatrix} 1 & & & 0 \\ & \alpha & & -\bar{\beta} \\ & & \ddots & \\ 0 & \beta & & \alpha \end{pmatrix} \cdot B$$

$$\begin{aligned} c_{kj} &= b_{kj} \cdot \alpha + b_{lj} \cdot \bar{\beta} \\ \Rightarrow c_{lj} &= -b_{kj} \cdot \beta + b_{lj} \cdot \alpha \\ c_{ij} &= b_{ij}, \quad i \neq k, l \end{aligned} \quad \left. \vphantom{\begin{aligned} c_{kj} &= b_{kj} \cdot \alpha + b_{lj} \cdot \bar{\beta} \\ c_{lj} &= -b_{kj} \cdot \beta + b_{lj} \cdot \alpha \end{aligned}} \right\} \begin{array}{l} \text{menjaju se elementi} \\ k\text{-te i } l\text{-te vrste} \end{array}$$

Želimo $c_{l,k-1} = 0$:

$$\begin{aligned} c_{l,k-1} &\stackrel{(*)}{=} -b_{k,k-1} \cdot \beta + b_{l,k-1} \cdot \alpha \\ &\stackrel{(**)}{=} -a_{k,k-1} \cdot \beta + a_{l,k-1} \cdot \alpha \\ &= 0 \end{aligned}$$

$$\Rightarrow \boxed{\beta = \frac{a_{l,k-1}}{a_{k,k-1}} \cdot \alpha}$$

$$\alpha^2 + |\beta|^2 = 1 \Rightarrow \alpha^2 + \left(\frac{a_{l,k-1}}{a_{k,k-1}} \alpha \right)^2 = 1$$

$$\alpha^2 \cdot \left[1 + \left(\frac{a_{l,k-1}}{a_{k,k-1}} \right)^2 \right] = 1$$

$$\alpha^2 = \frac{a_{k,k-1}^2}{a_{k,k-1}^2 + a_{l,k-1}^2}$$

$$\boxed{\alpha = \frac{|a_{k,k-1}|}{\sqrt{|a_{k,k-1}|^2 + |a_{l,k-1}|^2}}}$$

$$A = \begin{pmatrix} * & * & * \\ * & * & * \\ a_{31} & * & * \\ & a_{42} & * \\ a_{n1} & a_{n2} & * \end{pmatrix}$$

$$Q_{31} \rightarrow 0$$

$$C_{l,k-1} \rightarrow 0$$

$$l=3, \quad k-1=1 \\ k=2$$

$$U_{23}$$

$$Q_{42} \rightarrow 0$$

$$C_{l,k-1} \rightarrow 0$$

$$l=4, \quad k-1=1 \\ k=2$$

$$U_{24}$$

$$U_{n-1,n} \cdots U_{24} \cdot U_{23} \cdot A \cdot U_{23} U_{24} \cdots U_{n-1,n}$$

goranje - Hess.

$$\sim A$$

⊛ Ako je $a_{k,k-1} = 0$:

1) Ispod $a_{k,k-1}$ postoji neki $a_{m,k-1} \neq 0$
 izvrši se rotacija k -te i m -te vrste i kolone
 ($\alpha=0, \beta=1$)

2) Ispod $a_{k,k-1}$ nema ne-nula elemenata

$$\left[\begin{array}{ccc|cc} * & * & * & & * \\ * & * & * & & * \\ 0 & * & * & & * \\ \hline & & 0 & & * \\ 0 & & & & * \end{array} \right]$$